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EXTENDED SUPERSTRING AND UNIFICATION OF FOUR INTERACTIONS

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Abstract

First, the string and superstring are introduced. Next, some theories on particle physics are researched. Third, we propose the emergence string, which may derive the mass formulas of particles, etc. We study the extended superstring, which as a beautiful mathematical tool may be applied widely. Fourth, the unification of four interactions is searched. It includes the simplified supersymmetry. Fifth, we discuss neutrinos and their oscillation, in which some problems must be researched. Finally, some aspects of particle physics are discussed. String and superstring are the important method in physics.

Key words: particle physics, string, superstring, interaction, supersymmetry, neutrino

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I. String and Superstring

In order to explain success of quark model, but free quarks are not found, physicists proposed many models on quark confined, in which a famous model is string. It is proposed by Nambu and Nielson, et al., in 1969 based on the excited spectrum of double resonance on hadrons like string mathematically, all hadrons are the excited states of the fundamental entity string. Assume that the endpoint of the string is quark, so free quarks not exist. However, to overcome the ghost, space-time must be 10 or even 26 dimensions. The superstring as string is shown by the supersymmetry at high energy. The superstring is related to the differential topology.

In 1974 T. Yoneya, J. Schwarz, J. Scherk found independently that the low energy limit of string theory is the gauge theory of spin 1 and general relativity of spin 2, hence string may be a theory of quantum gravity, and a theory unifying gravity and other interactions. Now it has been found that a class of gauge theories can be regarded as string theory. Quark confinement in QCD can be considered due to the quark being pulled by the string formed by gluons, which is a way of quark confinement. The boundary condition for open string is that the normal derivative of the endpoint of string is zero, i.e., the Neumann boundary condition. This corresponds to requiring the endpoint of the string to move at the speed of light, so static string is impossible, at least it turns, and its lowest excitation state is the photon. In 1987, Green, Schwarz and Witten published book *Superstring Theory* [1].

Since 1994, solitons as solutions to the field equations have been central in string theory. Almost all objects, including the string, can be considered as solitons. At this point, in particular cases, as in the kink solution case, the Fermi number is even 1/2. In 1995 A. Strominger found the result of N. Seiberg-E. Witten in 1994, which can be used to explain the phase transitions between spaces with different topologies in the superstring. Such it may link completely different "vacuum" states. He used a particular soliton, a three-dimensional membrane.

In 1995, Witten incorporates 11-dimensional supergravity into string theory [2]. He shows that the mass is inversely proportional to the proportional coefficient of the charge and the coupling constant of the string, while the charge is independent of the coupling constant, and so the mass is inversely proportional to the coupling constant.

In 1995, J. Polchinski calculated for open string to illustrate some kind of dual on open string/closed string, and the ultraviolet in open string corresponds to the infrared in closed string, namely the later infrared/UV correspondence. Meanwhile, the classical interactions in closed string give the quantum effects in open string. Open string is dual to a closed string, s-t channel is dual, and a single circle graph with open string can be interpreted

as a tree plot with closed string. Almost all current strong-weak duals have supersymmetry [3].

Strings, especially superstrings, have great complexity, and also produce great arbitrariness. The cosmology has many metastable states, even no less than 10^{120}.

Smolin discussed three roads to quantum gravity: thermodynamics of black hole, the loop quantum gravity and the string theory [4]. High dimensions correspond to strong gravitation [4], and conversely, strong interactions are only high-dimensional gravity, such that gravitational interactions and strong interactions unify at high dimensions [5], namely three dimensions are gravitational interactions, and high dimensions corresponds to strong interactions, and small hidden space-time. String predicts the origin of the Big Bang and multiple universes.

If the Planck length becomes smaller than [4], the Planck time will also become larger accordingly. Both Planck length and time as invariants may obtain the Three Special Relativity [4]. This is three truncations introduced three invariants, in which no smaller time than t (p). And the superstring can be related to the dark energy [4].

In the theory of loop quantum gravity, the loop is the closed string. The rings are currently the kink, connection, winding, etc., in graphs, and particles are classified as the different winding modes [4]. This combines with differential equations can become dynamic, evolutionary maps, and may apply qualitative analysis theory, all kinds of singularities, especially focus sinks can form particles, stars, black holes, etc. The advantage of the loop is the set of equations that can be used in qualitative analytical theory. While string does not form systems of equations in general. But there should be systems of equations in high-dimensional space-time.

The lowest order vibration mode of open string corresponds to the photon s=1. The lowest order vibration mode of closed string corresponds to the graviton s=2. String theory has already the Feynman diagram.

Now string and superstring are the forefront and hot topics of theoretical physics [6]. This may product the Veneziano amplitude and the Regge pole, etc. Features of the superstring include: 1) higher (as 10) dimensions. 2) chirality due to rotation. 3) supersymmetry group SO(32), E(8)×E(8), O(16)×O(16). 4) non-divergence. 5) few free parameters, only string tension T, gauge coupling parameter g, and gravitational coupling parameter K, and K~ g^2T. 6) only single interaction, i.e., the string connected or disconnected. Continuous symmetry breaking derives the four interactions. 7) S-duality, T-duality, U-duality, etc., in which T-duality indicates a symmetry for the space size $R \to \alpha'/R$. The 10-dimensional superstring motion with time is namely 11 dimensions, and corresponds to 11-dimensional supergravity. 11-4=7,

Witten shows that the minimum dimension of a compact manifold having is 7 dimensions. If this corresponds to the particles, it may be quarks u,d,s and leptons v,e, μ and photon γ, or u,d,e(u,e,v) and their anti-particles (or supersymmetric particles) add single γ.

We proposed the velocity of light along the surface of a reel space in the higher-dimensional microscopic theory. In this case the velocity of light is not only variable, but also quantized on every side [7]. Further, if all 3 dimensions are extended to tubes, it will add 3 radii r, space will be 6 dimensions; if 3 dimensions are all spiral tubes, we should add 3 thread pitches h, then the space is 9 dimensions. The extra time is namely 10 dimensions.

2. Some Research of Particle Physics

In 1963 Newman-Unti-Tamburino derived the NUT solution in general relativity, which allow time travel, turn 360 degrees up stairs into the leaves at different levels of space-time.

Hiscock, et al., examined how quantization of matter fields might affect a particularly interesting subset of quasiregular: the "Taub-NUT-type" singularities [8]. These singularities are characterized by incomplete geodesics which spiral an infinite number of times around a topologically closed spatial dimension in a finite proper time, to end on one of a pair of null Cauchy horizons. In two-dimensional form ($x_2 = x_3$ =const), the coordinate transformation is:

$$y_0 = t\cosh x_1,\ y_1 = t\sinh x_1. \qquad (1)$$

They calculated the energy-momentum tensor for an untwisted scalar field on the closed Misner universe with three-torus topology ($T^3 \times R^1$). In this case, the points identified are:

$$x_1(t, x_1, x_2, x_3) \leftrightarrow (t, x_1 + na, x_2 + mb, x_3 + lc). \quad (2)$$

There are additional terms in the stress energy caused by the imposition of periodic boundary conditions in the x_2 and x_3 directions. This approach is more fundamental than the classical stability analyses.

In asymptotically (A)dS/flat space-times with NUT charge there is an additional periodicity constraint for t that arises from demanding the absence of Misner-string singularities. When matched with the periodicity β, this yields an additional consistency criterion that relates the mass and NUT parameters, the solutions of which produce generalizations of asymptotically flat Taub-bolt/NUT can classified by the asymptotically (A)dS case.

Mann, et al., considered the spherical Taub-NUT-dS solution [9], which is constructed as a circle fibration over the sphere in de Sitter background:

$$ds^2 = V(\tau)(dt + 2n\cos\theta d\phi)^2 - \frac{d\tau^2}{V(\tau)} + (\tau^2 + n^2)(d\theta^2 + \sin^2\theta d\phi^2), \quad (3)$$

where $V(\tau) = \dfrac{\tau^4 + (6n^2 - l^2)\tau^2 + 2ml^2\tau - n^2(3n^2 - l^2)}{(\tau^2 + n^2)l^2}$, (4)

and obtain mass

$$m = m_c = -\frac{\tau_c^4 - (6n^2 + l^2)\tau_c^2 - N^2(3n^2 + l^2)}{2l^2\tau_c}. \quad (5)$$

The metric of the Taub-NUT AdS solution in four dimensions:

$$ds^2 = -V(r)(dt - 2n\cos\theta d\phi)^2 + \frac{dr^2}{V(r)} + (r^2 + n^2)(d\theta^2 + \sin^2\theta d\phi^2), \quad (6)$$

where $V(r) = \dfrac{r^4 + (6n^2 + l^2)r^2 + 2ml^2r - n^2(3n^2 + l^2)}{(r^2 + n^2)l^2}$. (7)

Removal of the Misner-string singularities leads us to periodically identify the coordinate T with period $8\pi N / q$, where q is a non-negative integer.

In 1998 Santilli shown that the objections against the Einstein-Podolsky-Rosen (EPR) argument are valid for point-like particles in vacuum (*exterior dynamical systems*), but the same objections are inapplicable (rather than being violated) for extended particles within hyperdense physical media (*interior dynamical systems*) because the latter systems appear to admit an identical classical counterpart when treated with the isotopic branch of hadronic mathematics and mechanics. Now Santilli reviewed, upgraded and specialized the basic mathematical, physical and chemical methods for the study of the EPR prediction that quantum mechanics is not a complete theory. This includes basic methods [10], apparent proof of the EPR argument [11], and examples and applications, in which the validity of the EPR final statement is the effect that the wavefunction of quantum mechanics does not provide a complete description of the physical reality. The axiom-preserving "completion" of the quantum mechanical wavefunction due to deep wave-overlapping when represented via isomathematics, and shown that it permits an otherwise impossible representation of the attractive force between identical electrons pairs in valence coupling, as well as the representation of *all* characteristics of various physical and chemical systems existing in nature [12]. Moreover, Santill studied the classical determinism of EPR prediction by isomathematics [13].

From the EPR prediction, the nonlocality and entangled state become the frontline in modern physics. Based on the generalized Lorentz transformation (GLT) with superluminal in the complete special relativity [14,7], we proposed that the entangled states must obey GLT because they possess the

superluminal and some characters as the spacelike vectors. Further, it changes the phase of the entangled field, whose phase particle (phason) has some characters and corresponding equations. It is tachyon, and assume that it is similar to photon and J=1 and m=0 or mass is very small as similar neutrino, and may show the action at a distance. We researched that this field as wave has some characters [15]. We discussed the superluminal quantum communication by a pair of entangled states is generated on both positions, or by preparing and transmitting a pair of entangled instruments, so the superluminal quantum communication [15-17]. Manipulation for one position can pass the same message or information to the other, so we may implement the superluminal communication. Assume that the entangled field has a similar magnetic theory, which may be a quantum cosmic field, or be the extensive quantum theory, or God or the Buddha-fields and so on. These are all macroscopic fields, which correspond to de Broglie-Bohm nonlinear "hidden variable" theory, but it is microscopic [15]. In a word, study and application of nonlocality and entangled field have important scientific and social significance [15-17].

The length of the string is directly proportion the wavelength. String unifies wave-particle duality. It has energy-momentum and frequency-wavelength. Long waves correspond particularly clearly to fluctuations, while at small space-time and high energy a segment of string or it changes smaller, which shows hardly wave [14]. By relativity, the string is shortened at high speed. The string obeys the wave equation. For dynamic, high energy it constantly breaks into multiple bars and multiple quarks, and corresponds to the multiparticle production. Point particles are easier to show particle properties; string similar generalized objects, is not wave. Hadronic string contracted to point has tension and massless. It with tension corresponds to the Heisenberg self-interaction, and has length, and obtains mass. This is a point at low energy and a string at high energy. The scale of string is:

$$L_p = (G\hbar / c^3)^{1/2} \approx 10^{-33}\,\text{cm}, \quad L_p / L_{proton} = 10^{-20} \approx L_{atom} / L_{sun}. \quad (8)$$

So far the high energy experiments in the past sixty years have shown that the smallest mass fermions are proton, electron, neutrino and photon, which form the simplest model of particles. These fermions seem to be inseparable truth "atoms" (elements), because further experiments derive particles with bigger mass. They correspond to four interactions, and are also only stable particles. Such the final simplest theory is based on leptons (e-ν_e) and nucleons (p-n) or (u-d) in quark model with SU(2) symmetry and corresponding Yang-Mills (Y-M) field. Other particles and quark-lepton are their excited states. Their spectrum is mass formula and symmetric lifetime formula. Some applications are discussed. Further, the simplest interactions and unification of weak-strong interactions by QCD are searched. We research opposite

continuous separable models. Further, we proposed some possibly developed directions of particle physics, for example, violation of basic principles, in particular, the uncertainty principle, and precision and systematization of the simplest model, etc [18].

The spin-line velocity of superstring is $\omega(l/2)=c$, the potential energy $E=2lm_0$ is proportional to the length. The angular momentum is $M \propto E^2$. It is similar to the harmonic oscillator, and relates to the Regge trajectory, and is the development of dynamic integrators Veneziano model. The ground state mass of string is determined by the intercept constant in the Regge formula, the mass is proportional to the tension of string, the intercept is a function of the space-time dimension, usually is negative, so the mass square of the ground state of Bose string is negative, i.e., tachyon.

At present, Bagchi, et al., reconsider the tensionless limit on bosonic closed string theory, and tensionless path from closed to open strings [19]. Kaidi, et al., pointed out the classification of string theories via topological phases, and discussed the relationship between it and the K-theoretic classification of D-branes [20]. Costello, et al., discussed Chern-Simons Origin of Superstring Integrability [21]. Geyer, et al., proposed a procedure to determine the moduli-space integrands of loop-level superstring amplitudes for massless external states in terms of the field theory limit [22].

In quark-string model, the endpoints of string indicate the quark particles. Such mesons are composed of two quarks, and are unstable, corresponding string is not closed, and must be open string. Baryons are composed of three quarks, and can be closed string, such as ○, which can be stable. If it is developed as a multiple closure of concentric circles, this will correspond to an increase in the new quantum numbers S, C, etc. They are excited states with different radius.

The string theory is not problem in dotted model. The superstring uses an elliptic modular function, and corresponds to the B function and the Veneziano model. But it could not even get approximate particle mass spectrum like the earlier string model.

Guillen, et al., present a new method to explicitly compute string tree-level amplitudes involving one massive state and any number of massless ones. In this simplified model, they determined scattering massive string resonances through field-theory methods [23].

3. Emergence String, Extended Superstring and Their Applications

The string equation is developed to the wave equation, and corresponds to the oscillation- rotation model (ORM) [14] developed to the dynamical model. The emergence string just as the emergence particles behave as macroscopic phonons. S-duality of strong and weak interactions may correspond to string and particles, and according to Yukawa interaction and uncertainty principle, it

can turn into a scale duality, so the scale of string can be amplified. Early string is not diverge, and contains gravitons (J=2), D=26, and appears tachyon. New superstring has not tachyon, D=10, and eliminates anomaly.

Based on two moving states of the emergence string: oscillation and rotation, we derived its quantum potential and the equation, whose energy spectrum is the GMO mass formula and its modified accurate mass formula [24,14]:

$$M = M_0 + AS + B[I(I+1) - S^2/2]. \qquad (9)$$

These are some relations between the string and observable experimental data. Further, based on the Y-Q and I-U symmetries between mass and lifetime on the general SU(3) theory, we can derive the lifetime formulas of hyperons and mesons:

$$\tau = A[2U(U+1) - Q/2], \qquad (10)$$

and $$\tau = A'[(1/2) + 2U(U+1) - Q/2 - Q^2/3]. \qquad (11)$$

They agree better with experiments. It is a new method on lifetime of hadrons described by quantum numbers. They are symmetrical with the corresponding mass formulas, and can be unified for mass and lifetime. These formulas may also extend to describe masses and lifetime of heavy flavor hadrons [24-26].

The emergence strings of different levels can produce different strings. We may explain various macroscopic phenomena by the emergence string and the general extended string.

The Regge theory S=*aJ*+b, parity of mesons and baryons are positive and negative intersects. N and $\triangle$ also intersects appear. It constantly increases vector mesons and spins, and J^P change accordingly, and unify various hadrons and their resonances.

It is possibly related to some new extensive fields from complex number fields, in which matrices of 2m+1 and 2m+2 rank may have 4m elements.

The graph theory has already many applications [27]. Mathematically, we proposed developments in graph theory, which include five types of the basic elements: various solid lines (E), dotted lines (D) and wavy lines (W), which correspond to the edge-induced spanning-tree in some graphs, and vertices (V), fields (F). New extensive graph may describe some phenomena in natural science and social science. In particle physics, the solid lines represent fermions, the wavy lines represent photons, the dotted line represents meson, the double solid line represents the proton [28], and so on, which are namely Feynman diagrams. Vertices correspond to leptons of non-structure, fields corresponds to hadrons with internal structures. These lines may correspond to different interactions, the Regge poles and the Regge trajectories, strings, superstrings and bags, membranes, M theories, etc.

The baryons of three quark may be described by the Borromean rings (Fig.1) [29] with three loops in topology [30], here united they stand, divided they fall.

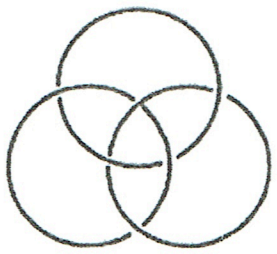

Fig. 1. Borromean rings

If we add directed, weighted and color for Borromean rings, it will be able to explain spins of particle and quarks with different masses and colors, etc.

Because the helix in three dimensional spaces and the basic formula of quantum mechanics are completely same, therefore, we proposed that the three-dimensional emergence string may form a whirlon (roton) model of particles, which may describe visually the basic relations of quantum mechanics [31]. The quark-leptons of different generations possibly correspond to whirlons with increasing helicity. This can be related to neutrinos as it particularly vividly describes neutrinos, and corresponds to parity nonconserved.

This may be related to the circle number of in topology. In 1990s the duality of superstring can be derived from the dual radius, whose reciprocal is equal to itself. This radius is the minimum scale in string theory, and is equal to the length scaling of string.

If whirlon corresponds to the particle spin, ν^{-1}=T will be the time required for a particle to rotate for one circle. The smaller the cycle T is, the greater the frequency, the faster, the higher the energy. Both superstring and whirlon are very geometrical [31].

The rotating string is 10-dimensional, and the lowest state of string is the tachyon. The string model shows that quarks are symmetric and dual resonances, which probably do not actually exist.

The whirlon corresponds to vector with both size and pitch, and vector is at four dimension macroscopic space-time. Whorl is also one of the eight treasures of Buddhism. It means that the Dharma sound of the Buddha is widely melodious, as far as the vast world.

Generally, the spiral motion is one of the basic motions for various macroscopic changes. It is described from Descartes to the helix motion in magnetic field, and pulsar and the spiral galaxies in astronomy. The

superstring (whirlon) is the most fundamental structural basis of the microworld. It is the result of a spatial curl with reduced dimension.

Now the N=4 supersymmetric Y-M theory is widely studied. Jiang, et al. discussed its planar exact three-point functions of determinant operators based on the gauge-string duality, and the proposal is put to the test at weak coupling [32]. Dorigoni, et al., obtained the novel representation of an integrated correlator in the SU(N) theory [33]. Gaberdiel, et al., proposed a description of worldsheet theory as a natural generalization of the tensionless string [34].

We discuss the like magnetic field and electromagnetic field of mesons. This can be quantized, and the general relativity is geometrical. Its energy levels combine string theory and integers and fractional Hell effects, etc. Assume that the quarks are connected by string, which is again the QCD and the gauge field, then the non-Abel fields and the QCD may lead to the string or like string. This string elongation requires force and energy, which determine the mass is proportional to length *l*, which is finite about 10^{-13} cm, and a finite range of action. Longer length corresponds to larger m, and easy to decay. They are pull fracture, i.e., different particles. QCD $\Leftrightarrow$ string=particles, both pay equal attention for the hypothesis of duality. Two different theories (e. g., weak and strong interactions) are equivalent. This is the development of the field and the particle. Wave-particle duality is its special example. Such a so different superstring theory will be a different manifestation of a larger theory. The M theory is namely a developed superstring theory. Further, superstring can be developed into a Yin-Yang double helix structure.

For a large class of black holes that can be implemented in string, string theory can in principle solve all quantum problems associated with black holes. Although the quantum physics of the most universal and simplest Schwarzschild black holes is not currently understood in string theory. The string theory shows that black holes, like any other thermodynamic system, obey quantum mechanics, and the entropy of black holes is also a measure of the number of microscopic states. The black hole string ranges from minimal to maximal, extremely heavy, i.e., the cosmic string. For the extremal black hole of string theory, there have charge and mass in 4 dimensions, and the charge reaches the maximum allowed. However, the temperature is near zero, about 10^{-7} K, and the entropy is not zero. In 1994, L. Susskind proposed a principle that any quantum system containing gravity satisfies the holography principle. Many physicists think always that in the quantum gravity theory, classical geometry should be replaced by similar concepts of noncommutation geometry, but which is not specific model.

The cosmic string is a huge emergence string, and is also an extended string. The string may have oscillation, rotation and twist, which corresponds to the torsion.

The string model must be high-dimensional. This is a one-dimensional space model, and may be developed into two and three dimensions, i.e., the brane model and the bag model. The latter is visually similar to the fireball and droplet model. The string is developed to a superstring model, which corresponds to have super-brane and super-bag models. The super-brane is an expansion of the superstring theory, and is a high-dimensional supersymmetric extension, i.e., the supersymmetric p-brane, and p=1 is the superstring. The quantization of string is only allowed if d=26 (boson string) or 10 (superstring) dimensions. Based on the cavity QED [35], we may research the cavity QCD, and combine the super-brane, etc. Further, we propose the tube-pipe particles, tube space and time, and tube physics, etc. Its difference with the string is that interior is empty. It corresponds to the bag.

The superstring has a supersymmetry. But it is minimal, so it may correspond to statistics. Further, the statistical properties of the superstring can be investigated. Its statistics should be linked to various statistical models. These are the possible big directions of superstring. The string model is phenomenal and more mathematical. Perhaps superstring should combine kinetics. Otherwise, superstring may only be a beautiful mathematical tool.

The problem of superstring may be in a high-dimensional space. The way out of high-dimensional space is probably that there are only new quantum space-time with micro small strength. Or appear in the multi-world, the negative matter [36-42] and the Yin-Yang world. Latter both are dual space-time. There has string, then the opposite string and the negative string and the negative opposite string may exist.

Santilli reviewed and upgraded the iso-representation of the spin 1/2 of nucleons according to the isotopic branch of hadronic mechanics, and is consistent with the spin J=1 and the magnetic moment of the Deuteron with the numeric value λ=2.65557 of Bohm hidden variable [43]. We researched some further possible directions of experiments. Based on the known mass and lifetime formulas and three generation hadrons, we studied differences on the electromagnetic mass of baryons and mesons, and their results are calculated for some hadrons [44].

Some structures in biology are helix. RNA is a single link structure on A-U and G-C. Here A is adenine and G is guanine, while U is uracil and C is cytosine. It is known that a helical line is:

$$x = a\cos t, y = b\sin t, z = dt. \qquad (12)$$

Here d is pitch. A form of the complex function is $z = a\cos t + ib\sin t$, $w = dt$. The double helix is:

$$z = a\cos(\varphi + \frac{C_1}{C_2}) \text{ and } w = d\varphi. \quad (13)$$

The whirlon [31] can combine the focus (source and sink). Radius $a(t)=b(t)= a_0 - 1$. $a= a_0$ for t=0; $a=0$ for t=1.

Based on the extensive quantum biology, we derived the double helical structure of DNA from Schrödinger equation, in which the basic quantum elements are A-T and G-C [45-47]. It is necessity mathematical conclusion that quantum mechanics has symmetry. Further, we researched the biological string [45], and string-brane theory and Calabi-Yau manifolds in biology [48].

4. Unification of Four Interactions, and Simplified Supersymmetry

For superstring, it first must study carefully that how it constitutes multiple particles and determines their different masses, namely, the relations of superstring and structure is a complex subject. Second, whether or not it still similar to the Yukawa interaction with exchange particles each other, or have already developed? Its theory is QCD, but whether or not should develop? This can be related to the development of weak interactions, etc. Although theoretically superstring passes through symmetry groups have already been connected to the unity of interactions.

Superstring may form the higher dimensional space. We assume that in 10-dimensional theory, 3 dimensional is gravitational field, and plus time is 4 dimensional general relativity; 1 dimensional is electromagnetic field. Both are the macroscopic fields. While the remaining 5 dimensions are microscopic fields, 3 dimension is strong field, and corresponds to SU(3); 2 dimension is weak field, and corresponds to SU(2). Such a microscopic space-time is 10 dimensions, and a macroscopic space-time is 4 or 5 dimension.

Group U(1) corresponds to electromagnetic interactions, (e,v) are electromagnetic and weak interactions, respectively. For m=0, they are Proca and Wely equations. QED has reached extremely high accuracy with perturbation. In atoms, especially hydrogen; in the nucleus it should be hydrogen, deuterium, tritium.

For the unified four interactions, the key is the unification of quantum theory and general relativity. This is also a dreams of final theory as the scientist's search for the ultimate law of nature [49].

From general relativity to quantum theory is the quantization and operators of the space-time quantity $G_{\mu\nu}$ and the matter quantity $T_{\mu\nu}$ in

$$G_{\mu\nu}\psi = aT_{\mu\nu}\psi. \quad (14)$$

We proposed possible equations unified between general relativity and quantum theory:

$$(\hat{G}_{kl} + \Lambda\hat{g}_{kl})\psi = k\hat{T}_{kl}\psi = \lambda_{kl}\psi, \quad (15)$$

Here λ_{kl} are eigenvalue of matrix. General case $\hat{R}_{kl}, \hat{T}_{kl}$ are operators. Therefore, 1) When they are simplified, theory should derive quantum mechanics and various corresponding equations. 2) They are not operator, theory becomes general relativity. 3) Microscopic wave property produces the wave function, in particular, it may express the microscopic gravitational field. 4) According to the correspondence principle, fourth dimension of tensor T is namely four dimensional momentum. 5) Probably operator is:

$$\psi = A\exp(iR_{\mu\nu}g_{\mu\nu}/\hbar), \tag{16}$$

or $$A\exp(iT_{\mu\nu}x_{\mu}x_{\nu}/\hbar), \tag{17}$$

therefore,

$$R_{\mu\nu} = i\hbar(\partial/\partial g_{\mu\nu}). \tag{18}$$

They combine usual quantization method of gravitational field, etc. 6) $\hat{R}_{kl}\psi = 0$ should be graviton equation [50,51].

From quantum theory to general relativity is a quantum theory in curved space-time. If gravity is only space-time curved, then the curved unification of space-time is the unified theory of four interactions, i.e., Eqs.(14).

In the quantum cosmic field the wave function of Universe obeys the Wheeler-de Witt equation:

$$(\hbar^2 G_{ijkl}\frac{\delta}{\delta g_{ij}}\frac{\delta}{\delta g_{kl}} + \sqrt{G^3}R)\psi(g) = 0. \tag{19}$$

Here $G_{ijkl} = \frac{1}{2\sqrt{G}}(g_{ik}g_{jl} + g_{il}g_{jk} - g_{ij}g_{kl})$. The equation may be developed to the nonlinear form:

$$(\hbar^2 G_{ijkl}\frac{\delta}{\delta g_{ij}}\frac{\delta}{\delta g_{kl}} + \sqrt{G^3}R - \hbar^2 G_{ijkl}\frac{1}{|u|}\frac{\delta^2|u|}{\delta g_{ij}\delta g_{kl}})u(g) = 0. \tag{20}$$

Further, in the extensive general relativity [52] any space-time of interactions and fields is all curved. Otherwise, the gravitational field is also quantized. These are both unifications.

SU(2) corresponds to weak interaction, QFD, and I=1/2 (p,n), (u,d) is strong interaction, SU(N) and QCD, Y-M equation. This has an asymptotic freedom. The accuracy should be improved.

Strong and weak interactions as short-range should be unified. Except different action ranges their main character is: strong interactions are attraction each other, and weak interactions are mutual repulsion and derive decay. We proposed a new method on their unification, whose coupling constants are negative and positive, respectively [53,54]. For QED and QCD,

beta functions have different signs. The electroweak unification theory has the same sign with QED.

In Fig. 2 three-generation quarks of the standard model are unified into a three-dimensional space:

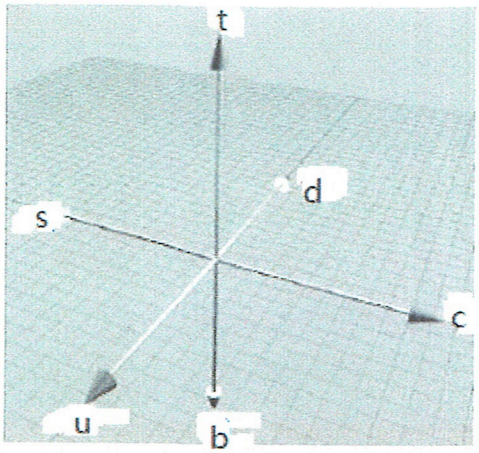

Fig. 2 three-generation quarks unified in three-dimensional space

This is developed to three-generation leptons (Fig.3):

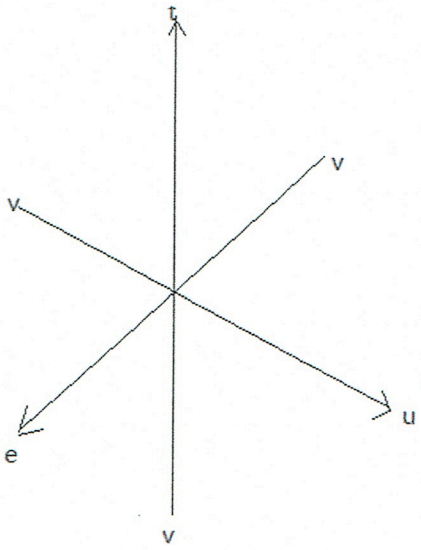

Fig. 3 three-generation leptons unified in three-dimensional space

Four interactions correspond to only four stable particles (photon, electron, neutrino ν_e and proton) [18], and four types: mass m, charge Q, lepton number and hadron number, where the baryon number is conserved, while the meson number is not conserved.

It is well-known that the theoretical base of particles is the standard model:

$$\begin{matrix} q=(2/3)e & -(1/3)e & -e & 0 \\ \begin{pmatrix} u \\ c \\ t \end{pmatrix} & \begin{pmatrix} d \\ s \\ b \end{pmatrix} & \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} & \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \end{matrix}. \qquad (21)$$

Such the second and third generations of quark-lepton are the different excited states of the first generation. The total charge number of per generation quarks is 3(2/3-1/3)=1, and the total charge number of per generation leptons is yet (-1+0)=-1. The total charge of per generation quark-lepton is 0. This is very symmetrical and beautiful theory [26].

We discussed the modified accurate mass formula, and applied to heavy flavor hadrons, and derived some predictions, m(Ω_{cc}^{+})=3950.7 or 3908MeV, and $m(\Xi_{bb})$=10396.8 or 10348.9MeV, etc. It is a quantitative and testable theory. Based on the new data, we proposed various lifetime formulas of heavy flavor hadrons, which very agree with experiments. This is a new method on lifetime of hadrons described by quantum numbers, and can be unified for mass and lifetime. Based on the known symmetrical particles and their excited states we proposed an approximate simplified supersymmetry theory, in which all bosons are the degenerate states [26].

Moreover, quarks have three generations, gluons should have three generations. Or second and third generation quarks are excited states of the first generation quarks, so the corresponding second and third generation gluons will also be the first generation excited states. This may be applied to astronomy and the universe.

5. Neutrinos, their Oscillations and New Research

In March, 2012 China Daya Bay neutrino study found that its speed is 10^{20} times less than the speed of light. This corresponds to the neutrinos having very small masses. The far 1500m electron neutrinos are 6% less than the near ones, which is usually attributed to neutrino oscillations.

If the neutrinos oscillate, then the symmetric three-generation leptons can also oscillate. Further, three generations of quarks can oscillate, possibly even with arbitrary particles.

This is more likely to the n-p oscillations in the nucleus, $n \to pe\bar{\nu}_e$ and conversely $p+e^{+} \Leftrightarrow n+\bar{\nu}_e$. $\mu \to e\nu_\mu\bar{\nu}_e$, and $\tau \to \mu\bar{\nu}_\mu\nu_\tau$.

According to the special relativity, neutrino has rest mass, so its velocity v<c. We may assume that different neutrinos have different velocities:

$$m_i = \frac{m_0}{\sqrt{1-(v_i/c)^2}}. \qquad (22)$$

1) If m is the same, so $v_i = c\sqrt{1-(m_0/m_i)^2}$, larger m the larger also velocity. 2) If v is the same, so m_i is proportional to m_0. The measured velocities of different neutrinos may determine 1) or 2).

The corresponding three-generation leptons can also be discussed similarly. Conversely, the mass formula of leptons $m = m_e(1+n/\alpha)$ [14] may be applied to three generations of neutrinos. Further, this may extend to three generations of quarks.

In June, 2001 Canada SNO released scientific results of detecting all three neutrinos from the sun, and confirm that solar neutrinos convert on their way to Earth, and the total flow of the three neutrinos fits well with the predictions of the standard solar model [55].

In order to explain the "solar neutrino missing" problem, the neutrino oscillation hypothesis is proposed. This must that neutrinos have rest masses (Dirac or Majorana masses), and they should be different for three types of neutrinos ν_e, ν_μ, ν_τ, and they are linear combinations of different masses $|\nu_1>, |\nu_2>$:

$$|\nu_e> = \cos\theta\,|\nu_1> + \sin\theta\,|\nu_2>, \qquad (23)$$

$$|\nu_\mu> = -\sin\theta\,|\nu_1> + \cos\theta\,|\nu_2>. \qquad (24)$$

Thus, the neutrino oscillation scale is:

$$L_0 = \frac{4\pi\hbar cE}{\delta m^2 c^4}. \qquad (25)$$

If the oscillation scale is about the distance between the Sun and Earth, the detection of ν_e from the Sun will be significantly reduced. If the oscillation scale is much larger than the distance between the Sun and the Earth, then the neutrino oscillation in the vacuum still cannot explain the solar neutrino disappearance. In 1998 Super K found ν_e-ν_μ oscillation, and in 2012 found ν_e-ν_τ oscillations [56].

Even if neutrinos oscillate, they cannot be missing. Neutrinos have masses, but smaller mass oscillations to larger masses do not conform to the basic energy conservation law. Unless it is a virtual oscillation, but the result is an actual neutrino missing! Or the three neutrinos have the same mass! General nuclear reactions will only produce ν_e. Only a large number of

occurrence μ,τ can produce ν_μ,ν_τ . The standard solar model is $4^1H \rightarrow {}^4He + 2e^+ + 2\nu_e$, and produce μ,τ and corresponding ν_μ,ν_τ.

We think that the oscillation of neutrinos with very different masses is difficult. According to the symmetry of leptons, three-generation neutrinos may not oscillate, but should be decay $\nu_\tau \rightarrow \nu_\mu \rightarrow \nu_e$, as $\tau \rightarrow \mu \rightarrow e$. Even if μ and electron e cannot oscillate, but only μ can decay to e.

Neutrino physics is not only related to lepton physics, and to a relativistic limit velocity, but also related to the Big Bang universe, dark matter and new physics [57], etc. According to the Yukawa interaction, massless neutrinos exchange with massive W-Z bosons, which is surprising. So far fourth leptons and $\tau \rightarrow \mu\gamma$ have not been discovered.

We must assume that the three lepton numbers are not conserved and change to each other. Some experiments and theories have been explored to this point. This is also related to the supersymmetric grand unified theory with family symmetry [58]. Gonzalez-Garcia and Nir discussed the phenomenon of neutrino masses and their mixing, the neutrino oscillations in vacuum and matter, and explore the existence of new physics, etc [59]. Delepine, et al., applied standard quantum field theory to calculate the CP asymmetry for neutrino oscillations and new physics, and the supersymmetry generalization of the standard model produces observable CP asymmetry in next generation neutrino experiments [60].

6. Discussion and Conclusion

For $\Omega^-(sss)$, it can assume: 1. there are three colors. 2. PEP is violated. 3. Three s quarks are in different excited states, such as 3 types of s are equidistant $m_0, m_0+\Delta, m_0+2\Delta$, so $m_0+\Delta$=557.48MeV.

In Feb. 2014, S-Kadif found electron-surrounded quantum droplets, which is similar particle with only a trillion of a second. This corresponds to high energy $\Delta E\Delta t = \hbar$, and $\Delta E = \hbar/\Delta t$, which probably violate PEP.

The strong interaction potential between the quarks changes to:

$$V = -\frac{4}{3}\frac{\alpha_s}{r} + Kr . \qquad (26)$$

For V=0, $r = \sqrt{\frac{4}{3}\frac{\alpha_s}{K}}$.

The general strong interaction potential is:

$$V = -\frac{\alpha_s}{r} + Kr . \qquad (27)$$

For V=0, $r = \sqrt{\frac{\alpha_s}{K}} \sim 10^{-13} cm$.

Fermions such as electron e can form a Cooper pair, so ee, qq, etc., may also form a pair, which correspond to bosons. If the force between fermions corresponds to weak interaction, it is also weak interaction between bosons. Furthermore, the positive and antimatter form a couple Q=0, and the positive and negative matters form a couple M=0 [36-42].

The three-color triquarks are baryons, and quark pair are mesons, both colored C=0. The color is introduced for PEP, and then is more complex gluons. The eight particles require mathematically the strongly interacting three-color SU(3), all have 0 mass and have the same strength g. Three generations of 6 quarks are beautiful, quarks can infer mass, etc. But, introduced color, 18 quarks and the corresponding theory are complexity. Three generations of three-color quarks are the SU(3)×SU(6) group. Three generations of leptons are exchanged by W-Z, and all eight particles have been found. And 18 color quarks and 8 gluons that should be the same for three generations are not found. The QCD and its ultraviolet slavery and asymptotic freedom seem just to prove that the free quark does not exist. (e, v) is the same generation I=1/2, the difference is only the electromagnetic interaction, which is similar to (p, n).

This is the spin-independent supersymmetry, e. g.

$$\begin{pmatrix} p & n \\ \pi^{\pm} & \pi^0 \end{pmatrix} I = \begin{matrix} 1/2 \\ 1 \end{matrix}, J = \begin{matrix} 1/2 \\ 0 \end{matrix}. \quad (28)$$

The open string is like magnetic charge, which may use the similar magnetic field theory of superstring, for motion, and string endpoint, etc. In quark-string model, this is similar to the magnetic monopole. Following the Dirac formula and its generalization $\mu e = n/2$. We can develop a like-monopole to a single quark, and obtain probably quark confinement. Further research will be the baryons of three quarks similar to the electromagnetic theory. The gluon force should be more similar to magnetic forces, without quarks correspond to no magnetic monopoles. There may be similar macroscopic closed string. Such magnetic monopoles and quarks will be simultaneously exist or absent. We can extend the magnetic monopole of Dirac theory, and obtain the fractional charge.

String is similar or as a topology, so it is difficult to describe quantitatively, but mainly a concept and mode.

Heisenberg proposed that a third constant should exist except c and h, which is probably scale r [61], and is related to string. The Planck space is

$l_P = \sqrt{\hbar G_N / c^3} = 1.616 \times 10^{-33} cm$ is too small, and it and α all are both combination constants.

In a word, from quantum mechanics to particle physics the applied mathematical methods are always a basic way. String and superstring as the new tool are very important aspect.

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